
Quadratic Functions

Exercise 7.3 - Modelling and Problem Solving

Question 8:

The total resistance R of two resistors R_1 and R_2 in parallel is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

A particular resistor is placed in parallel with one 20 ohms greater in resistance. The total resistance is 24 ohms. What are the resistances of two resistors?

Solution

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Re - writing the equation given:

$$\frac{1}{24} = \frac{1}{R_1} + \frac{1}{R_1 + 20}$$

Substituting $R = 24$ and

$$R_2 = R_1 + 20$$

$$\frac{1}{R_1} + \frac{1}{R_1 + 20} - \frac{1}{24} = 0$$

Re- arrange the equation
so the right side is Zero.

Writing the fractions with
the same denominator (L.C.D)

$$\frac{24(R_1 + 20) + 24R_1 - R_1(R_1 + 20)}{24 R_1 (R_1 + 20)} = 0$$

Expanding the brackets

$$\frac{24R_1 + 480 + 24R_1 - R_1^2 - 20R_1}{24 R_1 (R_1 + 20)} = 0$$

$$\frac{- R_1^2 + 28R_1 + 480}{24 R_1 (R_1 + 20)} = 0$$

Collecting like terms

$$- R_1^2 + 28R_1 + 480 = 0$$

Writing the numerator equal
zero

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Using the quadratic formula to
the equation above

$$a = -1, b = 28, c = 480$$

Stating values for a, b and c

Substituting the relevant
values into the formula

$$x = \frac{- 28 \pm \sqrt{(28)^2 - 4 \times -1 \times 480}}{2 \times -1}$$

$$x = \frac{-28 \pm \sqrt{2704}}{-2}$$

$$x = \frac{-28 \pm 52}{-2}$$

$$x = \frac{-28 + 52}{-2}$$

$$= -12$$

Negative resistance does not
make sense

OR

$$x = \frac{-28 - 52}{-2}$$

$$= 40$$

So:

$$R_1 = 40 \text{ ohms}$$

$$\begin{aligned} R_2 &= R_1 + 20 \\ &= 40 + 20 \\ &= 60 \text{ ohms} \end{aligned}$$

Using the information given
Substituting 40 for R_1

Answer: The resistance of the two resistors are $R_1 = 40$ ohms and
 $R_2 = 60$ ohms.
