Differentiation

Exercise 8.4 – Question 2

2i)

Differentiate $(-9x^{-5} + \frac{1}{4x})$

Solution

Using $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x))$

So:

$$\frac{d}{dx}(-9x^{-5} + \frac{1}{4x})$$

$$= \frac{d}{dx}(-9x^{-5}) + \frac{d}{dx}(\frac{1}{4x})$$

$$= \frac{d}{dx}(-9x^{-5}) + \frac{d}{dx}(\frac{1}{4}x^{-1})$$

Now Using $\frac{d}{dx}(kx^n) = kx^{n-1}$

$$= 45x^{-6} + \frac{1}{4} \times -1 \times x^{-2}$$

$$= 45\ x^{-6} - \frac{1}{4x^2}$$

Answer:

$$\frac{d}{dx}\left(-9x^{-5} + \frac{1}{4x}\right) = 45x^{-6} - \frac{1}{4x^2}$$

Exercise 8.9 - Modelling and Problem Solving

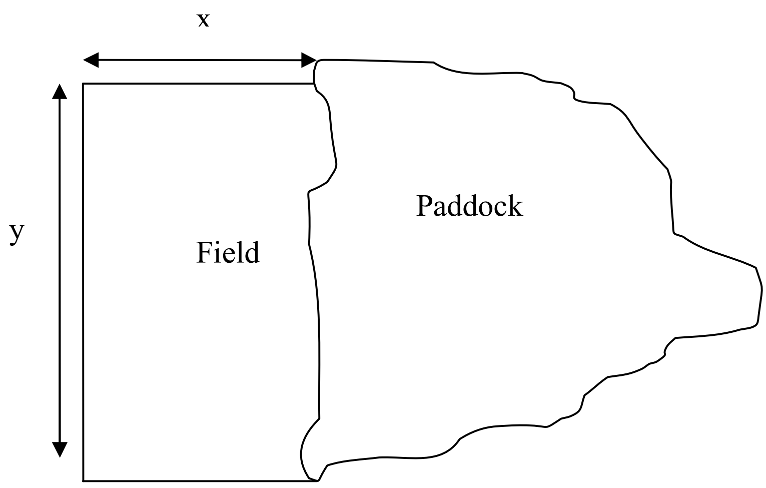
Question 10:

A rectangular field shares one side with an existing paddock and so requires no fence. There is only 1000m of fencing material to fence the remaining sides. Find the maximum possible area of the field.

Solution

In this problem we want to maximize the area of a field and we know that will use 1000m of fencing material. So, the area will be the function we are trying to optimize and the amount of fencing is the extra information that will be used to write the function in terms of only one variable

Let
x = Width of the field
y = length of the field
A = area of the field



So
$A = x\,y$

$2x + y = 1000$ Using the information given

$y = 1000 - 2x$ Making y the subject of equation

$A = x(1000 - 2x)$ Substituting the value of y into the function of area

$A = 1000x - 2x^2$

Finding the maximum value for A:

$A'(x) = 1000 - 4x$ Writing the derivative

$1000 - 4x = 0$ Setting the derivative equal to zero

$-4x = -1000$

$x = 250$ m

$y = 1000 - 2x$

$y = 1000 - 500$ Substituting $x = 250$
$= 500$

(250, 500) is a turning point.

The maximum of the function of the area occurs at the turning point. So, substitute $x = 250$ into the function

$A = 1000\times 250 - 2(250)^2$
$= 125000 \text{ m}^2$

Answer: The maximum possible area of the field is 125000 m^2 when it is 250m wide and 500m long.